LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – November 2009

MT 3501 - ALGEBRA, CAL. & VECTOR ANALYSIS

Date & Time: 4/11/2009 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

SECTION – A

Answer ALL questions **(10 × 2 = 20)**

1. Evaluate $\int_0^a\int_0^b\int_0^c x\,dx\,dy\,dz$.
2. If $x = u(1+v);\ y = v(1+u)$, find $\frac{\partial(x,y)}{\partial(u,v)}$.
3. Find the partial differential equation by eliminating the constants in $x = (x + a)(y + b)$.
4. Find the partial differential equation by eliminating the arbitrary function from $z=f(x^2 + y^2 + z^2)$.
5. Show that the vector $3x^2\hat{i} - 4xy^2\hat{j} + 2xyz\hat{k}$ is solenoidal.
6. If $\phi(x, y, z) = x^2y + y^2x + z^2$, find $\nabla\phi$ at (1, 1, 1).
7. If $L(f(t)) = F(s)$, What is $L(e^{-at}f(t))$?
8. Find $L^{-1}\left(\frac{F(s)}{s}\right)$.
9. Define Euler's function $\phi(N)$.
10. Find the remainder when 2^{1000} is divided by 17.

SECTION – B

Answer any FIVE questions **(5 × 8 = 40)**

11. Evaluate by change of order of integration in $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$.
12. Evaluate $\int_0^{\infty} e^{-x^2} dx$.
13. Find the complete and singular solution of $z = xp + yq + p^2 - q^2$.
14. Solve $p^2z^2 + q^2 = 1$.
15. Prove that $\bar{F} = (y^2\cos x + z^3)\hat{i} + (2y\sin x - 4)\hat{j} + 3xz^2\hat{k}$ is irrotational and find its scalar potential.
16. Find $L(\sin^4 t)$.

17. Find $L^{-1}\left[\frac{1+2s}{(s+2)^2(s-1)^2}\right]$

18. Find the highest power of 3 dividing (1000)!

SECTION – C

Answer any TWO questions **($2 \times 20 = 40$)**

19. (a) Evaluate $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ taken over the volume bounded by the planes $x = 0; \; y = 0; \; z = 0; \; x + y + z = 1$.

(b) Show that $\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$

20. (a) Solve $p^2 + q^2 = z^2(x + y)$

(b) Solve $(3z - 4)p + (4x - 2z)\, q = 2y - 3x.$

21. (a) Verify Gauss theorem for $\bar{F} = x\hat{i} + y\hat{j} + z\hat{k}$ taken over the region bounded by the planes
$x = 0; x = a; y = 0; y = a; z = 0; z = a$.

(b) Find $L^{-1}\left[\frac{s-3}{s^2+4s+13}\right]$ (12 + 8)

22. (a) Solve the differential equation $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} - 5y = te^t$ using Laplace transformation given $y(0) = y'(0) = 0$.

(b) Show that the 8^{th} power of any number is of the form $17m$ or $17m \pm 1$.

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